

**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**



**B.Sc. DEGREE EXAMINATION – MATHEMATICS**

**FIFTH SEMESTER – APRIL 2023**

**UMT 5503 – DISCRETE MATHEMATICS**

Date: 05-05-2023

Dept. No.

Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

**PART A**

**Answer ALL the questions**

**(10 x 2 =20)**

1. Construct the truth table for  $(P \vee Q) \wedge \neg(P \wedge Q)$ .
2. Write the following statement in symbolic form:  
“If either Jerry takes calculus or Ken takes sociology then Larry will take English”.
3. Show that the conclusion  $C: Q$  follows from the premises  $H_1: \neg P$  and  $H_2: P \vee Q$ .
4. Let  $A = \{1,2,3,4,5,6\}$ . Determine the truth value of  $(\exists x \in A)(x^2 > 25)$ .
5. Define semigroup and give an example.
6. Prove that every cyclic monoid is commutative.
7. Draw Hasse diagram for  $(\rho(A), \subseteq)$  where  $A = \{a, b, c\}$ .
8. Is the poset  $(\mathbb{Z}^+, /)$  a lattice? Justify it.
9. Let  $(B, +, \cdot)$  be a Boolean algebra and if  $a, b \in B$ , prove that  $a + ab = a$ .
10. Define Boolean algebra.

**PART B**

**Answer any FIVE questions**

**(5 x 8 =40)**

11. Prove that  $(\neg P \wedge (\neg Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R)) \Leftrightarrow R$ .
12. Obtain CNF of  $\neg(P \vee Q) \Leftrightarrow (P \wedge Q)$ .
13. Establish the validity of the following argument: “If an integer is divisible by 10 then it is divisible by 2. If an integer is divisible by 2 then it is divisible by 3. Therefore, the integer divisible by 10 is also divisible by 3”.
14. Show that the composition of semigroup homomorphism is also a semigroup homomorphism.
15. Prove that for any commutative monoid  $(M, *)$ , the set of idempotent elements of  $M$  forms a sub monoid.
16. Prove that any lattice homomorphism is order preserving.
17. If  $S_{42}$  is the set of all divisors of 42 and  $D$  is the relation ‘divisor of’ on  $S_{42}$ . Prove that  $(S_{42}, D)$  is a complemented lattice.
18. State and prove Stone’s Representation theorem.

## PART C

Answer any TWO questions

(2 x 20 =40)

19. (a) Determine whether  $(\neg Q \wedge (P \rightarrow Q)) \rightarrow \neg P$  is a tautology.

(b) Obtain the PDNF and PCNF of  $(\neg P \rightarrow R) \wedge (Q \rightleftharpoons P)$ . (8+12)

20. (a) Show that  $S \vee R$  is tautologically implied by  $P \vee Q$ ,  $P \rightarrow R$  and  $Q \rightarrow S$ .

(b) Let  $(S,*)$  be a semigroup and  $R$  be a congruence relation on  $(S,*)$ . Then prove that the quotient set  $S/R$  is a semigroup  $(S/R, \oplus)$  where the operation  $\oplus$  corresponds to the operation  $*$  on  $S$ . Also show that there exists a homomorphism from  $(S,*)$  onto  $(S/R, \oplus)$ . (8+12)

21. State and prove idempotent, associative, commutative and absorption properties of Lattice.

(20)

22. (a) In a complemented distributive lattice, show that the following are equivalent

$$a \leq b \Leftrightarrow a \wedge b' = 0 \Leftrightarrow a' \vee b = 1 \Leftrightarrow b' \leq a'.$$

(b) Obtain the product-of-sums canonical forms of the Boolean expression in three variables  $x_1, x_2, x_3$

(i)  $x_1 * x_2$

(ii)  $x_1 \oplus x_2$

(iii)  $(x_1 * x_2)' \oplus x_3$ .

(10+10)

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